Statistical Science

Chapter 2.2 Types of Measurement Scale

ReCap (Ch 1, Ch 2.1)
Ch2 Quantities
2.1 Five part definition
2.2 Types of measurement scale
2.3 Data collection, recording, and error checking
2.4 Graphical and tabular display of data
Critique of graphs and tables (Lab 5)
2.5 Ratio Scale Units
Base
Standard Multiples
Commonly used units
2.6 Dimensions

on chalk board

Not here last time?
Course Outline
Questionnaire results

Discussion of Cards Lab:
Anybody come up with "wrong" rule that works?
In 1997 mutually exclusive pairs introduced ("test" cards), before going to multiple working hypotheses ("crucial" cards).
Ask for discussion of this, comparison of "test" and "crucial."
In 1998 crucial cards only.

ReCap Chapter 1

Statistical science is not applied mathematics;
it is a science that draws inferences from explanatory models based on measurement.
Statistical models consist of an explanatory model and an error model.
Explanatory models, in symbolic form, are developed from verbal models.
△ Verbal, graphical, and formal model (equations)
Models are used to make: useful calculations (species extinction),
quantify evidence (likelihood ratios),
quantify uncertainty (p-values).
Role of statistics: Development of models (exploratory analysis)
Formal evaluation of models (confirmatory analysis)
Statistical science is a course in quantitative reasoning.
It is not a course in rote learning of list of tests.
It is a course in how to apply statistics to measured quantities.
It will integrate models with statistics.

ReCap Chapter 2.1

Models express ideas about the relation of quantities.
Quantities are defined in 5 parts.

Wrap-up

Measurements are made on many types of scales.
The most common are nominal, ordinal, interval, log ratio, and ratio scales.

Quizzes are weekly because one component of learning is active engagement with ma
Quizzes are second half of a lecture, one day a week.
Collaboration is encouraged in labs, in exercises, in note taking.
    For example, use e-mail to exchange data sets.
Collaboration is not encouraged on quizzes and exams.
    These are individual efforts.
    Set-up of the room a problem (sitting adjacent)
    So: Two versions of exams
        Quiz today: Not collaborative. Open book. (just like exams)

I hand out the quiz as quickly as possible, 10-20 minutes before end of class.
To speed it up, someone from class helps me.

Types of Measurement Scale (Schneider 2009 *Quantitative Ecology* Chapter 3.5)

In 1946 the psychologist S.S. Stevens (*Science* 103:677) distinguished 4 types of measurement scale.

Nominal. Outcome of measurement is "yes" or "no" also coded as 0 or 1

Ordinal. Outcome is ranking: 1st, 2nd, 3rd, etc.
There is no information about magnitude of difference from one rank to next.

Interval. The distance between measurements is known, but zero is an arbitrary point.
Addition and subtraction are valid. Multiplication and division are not.
Example of compass direction. Direction is relative to an arbitrary point.
Other examples are Dates (2 January, Julian day 180)
Latitude/longitude
Indices are often on an interval scale.
An index of relative abundance might have an arbitrary minimum of zero.
This eliminates zeros due to chance and zeros due to unsuitable habitat.

Ratio. Distance between measurements is known, zero point is absolute, not arbitrary.
An example is degrees Kelvin. There is no heat content at 0^{o} K
In contrast, 0^{o} C does not mean "no temperature."
Molecular motion (heat) is still present. So 0^{o} C is interval.
Multiplication and division are valid.
For example, a population growing at $r = 5\%$ year^{-1} has
twice the intrinsic rate of increase as a population with $r = 2.5\%$ year^{-1}
In contrast, twice 20 January (Julian date 20) has no meaning.
Twice 10 degrees Celsius is not the same twice 10 degrees Fahrenheit.

Types of Measurement Scale - Extensions

In 1959 Stevens expanded the typology to include log-interval and absolute scales.

Log interval scale:

Numbers are assigned so that ratios of values reflect ratios in the attribute being measured

Examples: earthquake intensity (Richter scale) and acidity pH.

Absolute scale;

No units assigned

Example: Counts

According to Stevens typology, counts are not ratio scale.

However, counts have an absolute zero.

Counts can be assigned units. Counts with units can be multiplied.

(2 hawks)(4 sparrows) = 8 potential predator-prey contacts.

(4 deer)(3 days) = 12 deer-days, a measure of impact on forage.

(3 deer)(4 days) = 12 deer-days, same impact as above.

For practical purposes, counts are readily treated as ratio scale.

Notes on the NOIR typology

Ratio compared to interval type of measurement scale.

Ratio scale time (your age) versus interval scale time (calendar date)

Ratio scale length (lake diameter) versus interval scale (lat/long).

Interval scale measurements can often be converted to ratio scale by taking a difference.

For example, 45^{o} means NorthEast, it is interval scale.

A sailboat turns 45^{o} (from 0^{o} to 45^{o}, from 30^{o} to 75^{o} , etc)

The difference in direction is on a ratio scale.

All four types are commonly encountered in both the natural and social sciences.

Ratio and interval measurements are sometimes grouped together as <u>cardinal</u>, leading to 3 categories: Nominal, Ordinal, Cardinal.

Nominal, ordinal, interval, and ratio scales differ in the amount of information.

Many 'non-parametric' tests reduce interval or ratio scale to a less informative rank scale and so can give different results than using the interval or ratio scale data for analysis.

The NOIR typology is useful.

The type of measurement scale indicates the mathematical operations that can be performed on a variable (Table 2.2.1). The mathematical operations determine allowable descriptive statistics -- measures of central tendency and dispersion. Type of measurement scale is a useful guide to appropriate inferential models, measures of fit of data to models, and error models.

Table 2.2.1.Variable type indicates allowable operations, descriptive statistics, explanatory models, and error distributions. Link functions are shown for error distributions.

	Nominal	Ordinal	Interval	Ratio
Operation				
Equality	Y	Y	Y	Y
Order		Y	Y	Y
Add/Subtract			Y	Y
Muliply/Divide				Y
Central tendency				
Mode	Y	Y	Y	Y
Median		Y	Y	Y
Arithmetic mean			Y	Y
Geometric mean				Y
Dispersion				
Range		Y	Y	Y
Standard deviation			Y	Y
Variance			Y	Y
Linear models				
Likelihood ratio			Y	Y
Test statistics: *t,F,G*			Y	Y
Log-linear models				Y
Power laws				Y
Probability, Odds				Y
Generalized Linear Model	Y	Y	Y	Y
Distributions (error models)				
Binomial	Logit	Logit		
Ordered Multinomial		Logit		
Normal			Identity	Log
Gamma			Identity	Log
Beta binomial	Logit		Identity	Log

These four types of scales are useful in understanding differences among statistical procedures. Examples:

ANOVA is based on nominal scale explanatory variable (classes).
Regression is based on interval or ratio scale explanatory variable.
Logistic regression applies to nominal scale counts (units scored Y/N)
Poisson regression applies to ratio scale counts (counts/unit)

References

Stevens, S. S. (1946). On the theory of scales of measurement. *Science* 103: 677–680. This article was published in a widely distributed journal, to address the claim (made by some physicists) that presence/absence, or ranks, were not legitimate types of measurement.

Stevens, S.S. Measurement and man 1958. *Science* 127:383-389

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Critiques

Velleman, P.F. and Wilkinson, L. (1993) Nominal, ordinal, interval and ratio typology are misleading. *The American Statistician*, 47: 65-723

Moscati, I 2018. Stevens and the operational definition of measurement in psychology, 1935–1950. Chapter 8 Pages 139–146 In: *Measuring Utility: From the Marginal Revolution to Behavioral Economics*. Oxford Studies in the History of Economics (New York: Oxford University Press).

Review

Brunsdon, C. 2018. Quantitative methods III: Scales of measurement in quantitative human geography. *Progress in Human Geography* 42: 610–621