Statistical Science

Chapter 3.2 Operations on Ratio Scale Quantities

ReCap. Quantitative reasoning(Ch 1)
Quantities (Ch2)
Re-Scaling (Ch3)
3.1 Logical Re-scaling
3.2 Operations on Ratio Scale Quantities
3.3 Descriptive Statistics and Rescaling
3.4 Unit Conversion and Rigid Rescaling

on chalk board

```
Not here last time?
         Syllabus
```

Recap Chapter 1
Quantitative reasoning: Example of scallops, which combined stats and models
Recap Chapter 2
Quantities: Five part definition
Measurements made on four types of scale: nominal, ordinal, interval, ratio
Recap Chapter 3. Re-scaling
Logical rescaling (from one type of unit to another).
Re-scaling is a common technique in quantitative biology.

Today: Operations on Ratio Scale Quantities
Rescaling ratio scale units, looking first at permissible operations.
These will be visualized, rather than treated as abstract rules.
Then based on these operations the concepts of normalization,
rigid rescaling, and elastic rescaling will be presented.

Wrap-up:
Operations on measured quantities differ from operations on numbers.
- -the rules differ
- -physically interpretable, not just abstract mathematical procedures

Examples of physical interpretation of the operations of addition, subtraction, multiplication, and division.

Rescaling ratio scale quantities—Operations (Schneider 2009 Chapter 5.3)

Operations on measured quantities differ from operations on numbers.
- the rules differ
- physically interpretable, not just abstract mathematical procedures

Concept 1. Rules for operations on scaled quantities (addition, division, etc) differ from rules for operations on numbers.

Example of same units 1 metre + 2 metre
Example of similar units: yards and metres Similar units belong to the same dimension
Example of dissimilar units: metres and years Dissimilar units belong to different dimensions

Concept 2. Allowable operations depend on whether quantities have
- same units,
- similar units (same dimension),
- or different units (different dimension).

Table 3.2. Operations on ratio scale units.

Operation		Example	Same	Similar	Dissimilar	Interpretation
+	add	1L + 2L	Yes	←(convert)		sum
+	add	1L + 2T			No	
−	subtract	1L − 2L	Yes	←(convert)		difference
−	subtract	1L − 2T			No	
*	multiply	1L · 2L	Yes	←(convert)		Interact, stretch
*		1L · 1T			Yes	new units
÷	divide	2L ÷ 1L	Yes	←(convert)		normalize
÷	divide	2L ÷ 1T			Yes	new units
exp	exponentiate	$1L^{1T}$	No	No	No	
exp	exponentiate	$1L^{n}$	Yes	Yes	Yes	elastic rescaling
log	logarithm	$\log_{1L}(1T)$	No	No	No	
log	logarithm	$\log_{n}(1T)$	Yes	Yes	Yes	elastic rescaling

Concept 3. Operations on scaled quantities are usually interpretable and visualizable.
- add time units (wait longer)
- add counts (population growth)
- increase area occupied via random motion (diffusion)
- add velocities (accelerate).

```
Fill in table from top down, one
line at a time, examples follow.
```

Operations–additions and subtraction

The operations of addition and subtraction correspond to physically interpretable actions. Physical removal or addition (think of dumping two quarts of water into a large container) can be represented by a simple mathematical operation.

0.946 litre + 0.946 litre = 1.892 litre

Only quantities with the same units can be added and subtracted.

4 Joules + 6 Joules = 10 Joules

4 Joules + 6 kcal = ?

Joules and kcal are similar, both are energy.
Joules and kcal are not the same, and cannot be added.

Quantities with similar units can be added or subtracted after conversion to the same units.
Quantities with different units cannot be added or subtracted

The rules for units distinguish similar from dissimilar units; this in turn depends on how the units have been defined. For example, apples and oranges are not similar units; we cannot add them. We can, however, define a new unit 'fruit' that allow one group of fruit (all apples) to be added to another group (all oranges).

Operations--Multiplication

Quantities with the same dimensions can be multiplied. This changes the exponent of the units. This creates a new unit. Interpretation depends on the units.

$L \cdot L = L^2$ stretching. Think of sweeping sticks at right angles to make areas.
$L^2 \cdot L = L^3$ stretching. Sweep out volume by pulling a plane sideways.
$\# \cdot \# = \#^2$ pairwise interaction of entities
$L/T \cdot L/T = L^2T^{-2}$ energy release or gain, via acceleration or change in frequency.

Example: Euclidean stretching $1\ m \cdot 1\ m$ (pulled sideways) $= 1\ m^2$
$10\ m^2 \cdot 2\ m$ (pulled sideways into cube) $= 2\ m^3$

Example: fractal stretching $10\ m^1 \cdot 10\ m^{1.2} = 10\ m^{2.2}$

A crooked line)$m^{1.2}$) has been stretched into a crooked area ($m^{2.2}$).

Quantities with similar units are first converted to the same unit, then multiplied.

Quantities with dissimilar units can be multiplied. This generates a new unit.
For example, multiplying 4 ants times 3 days residency results in 12 ant-days. This new unit is a measure of potential for interaction with some environmental factor, such as food. Expect 4 ants over 3 days to have similar food consumption as 12 ants for 1 day.

Operations--Division

Quantities with the <u>same</u> unit can be divided. This results in unitless (nondimensional) ratios.

Example: scaling (how many 10m by 10m blocks in a hectare ?)

$$1 \text{ ha} / (10\text{m})^2 = (100\text{m})^2/(10\text{m})^2 = 10000 \text{ m}^2 / 1000 \text{ m}^2 = 10$$

Quantities with <u>similar</u> units can be divided; this results in a conversion factor that is a dimensionless ratio.

Example: a square kilometre is similar to a hectare, but 100 times larger:
$km^2/ha = 100$. The ratio of these units has no units.

Example: metre/yard = 0.9144

Quantities with <u>dissimilar</u> units can be divided. This results in new units, because the ratio of dissimilar units will have units and dimensions.

$km^2/km = km$
Joule/second = Watt.

This stands in contrast to the ratio of similar units, a ratio that is a number with no units.

Summary

In science we work with scaled quantities, not with numbers.
Working with scaled quantities differs from working only with numbers.
Erroneous calculation result if we ignore units
For example: Using numerical ranks as if they were ratio scaled quantities.

Your turn.

List a quantity of interest to you (with units). ________________
List a value a second quantity of interest to you. ________________
Is the ratio of one to the other an interpretable quantity? ________
If so, name the quantity and show its units. ________________
If not, name two quantities where the ratio is interpretable

____
____

Name the ratio and show the units ___________ ________________